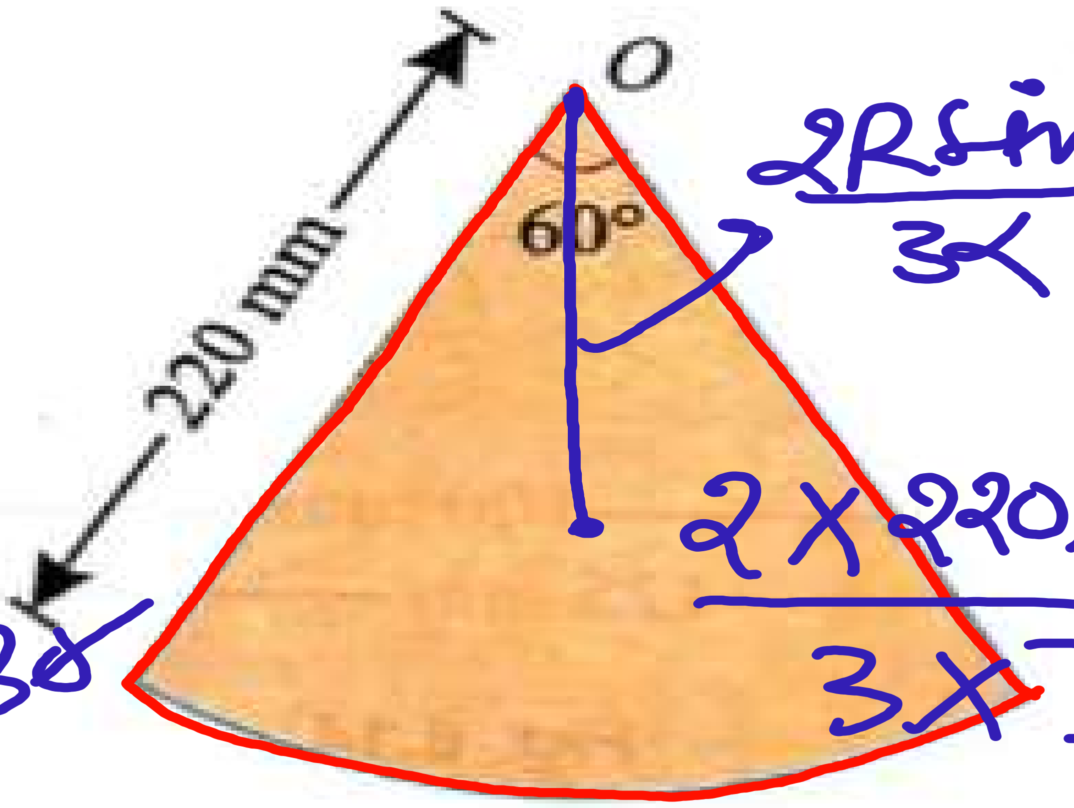
A plane lamina of 220 mm radius is shown in figure given below.

$\alpha = 30^\circ$

$\alpha = 30^\circ$

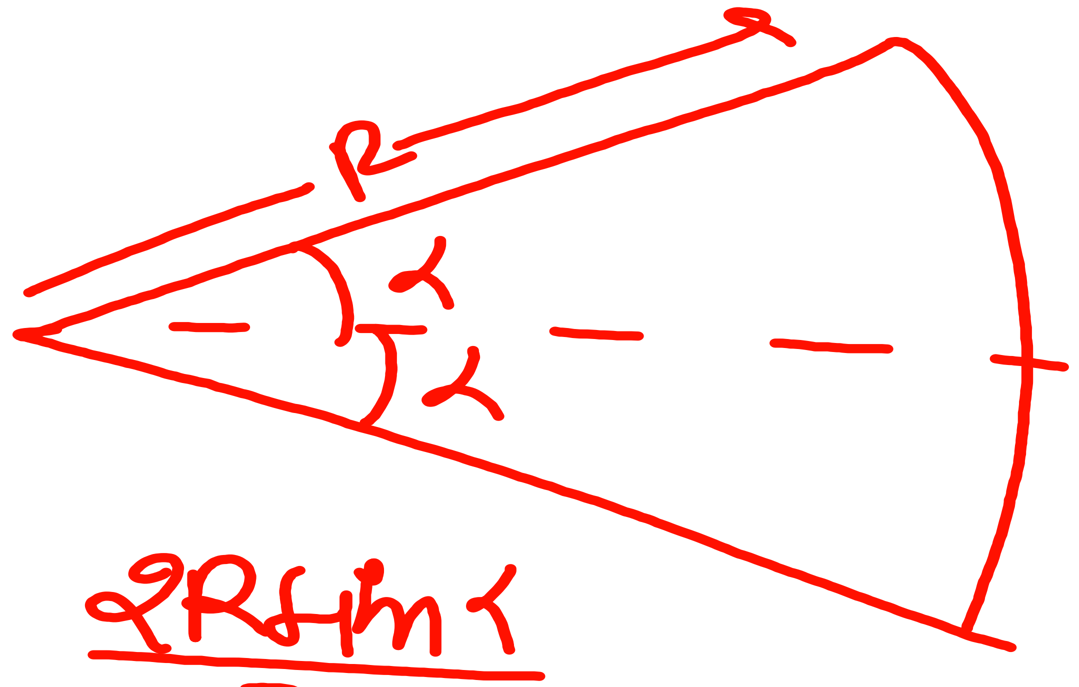
$\pi^c \to 180^\circ$

$1^\circ = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$



$\frac{2R\sin\alpha}{3\alpha}$

$\frac{2 \times 220 \times \frac{1}{2}}{3 \times \frac{\pi}{6}}$



$\frac{2R\sin\alpha}{3\alpha}$

$\frac{\pi}{6}$ Find the centre of gravity of lamina from the point O.

$\bar{x} = ?$

$A_1 \to \pi r^2$ ✓

$$\bar{X} = \frac{A_1\bar{x}_1 - A_2\bar{x}_2}{A_1 - A_2}$$

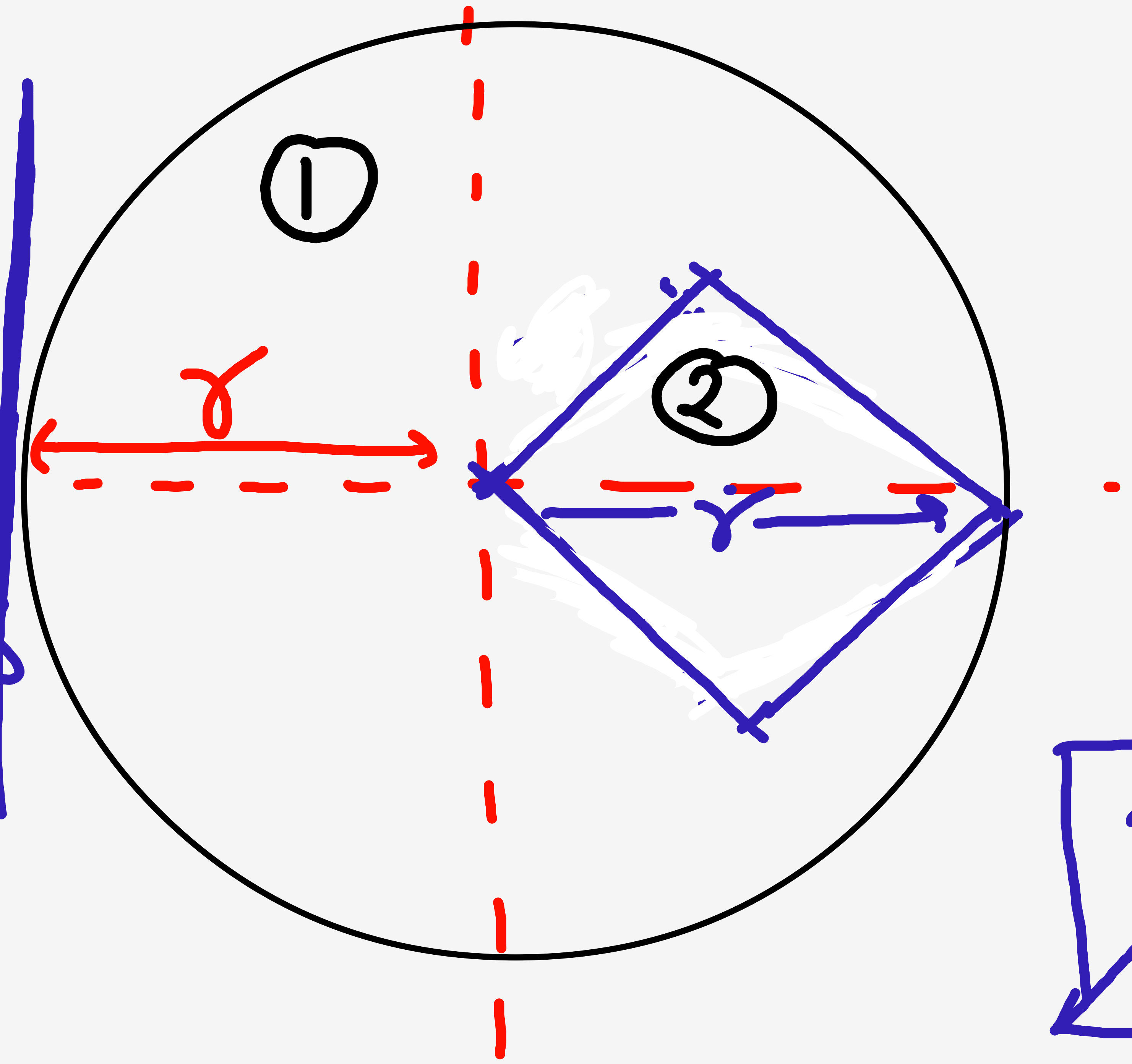
$\frac{1}{2}r^2 \times \frac{3}{2} r$

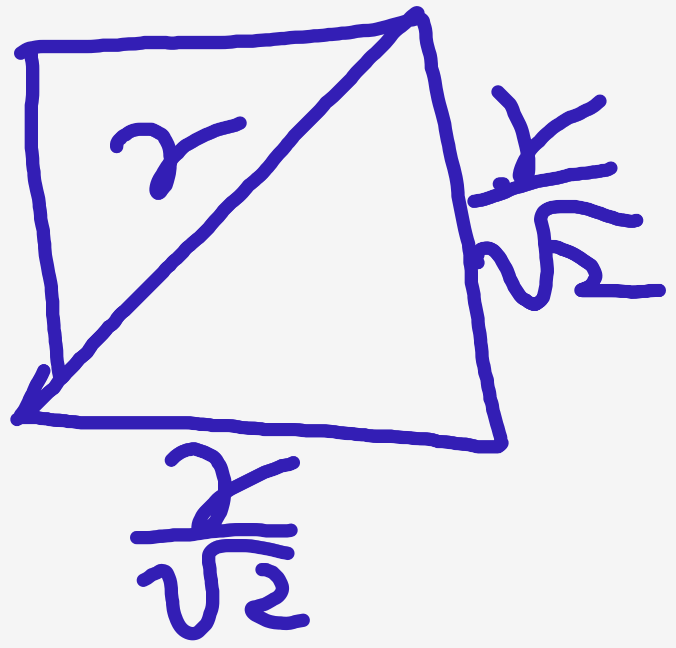
$\frac{3}{4}r^3$

$\bar{x}_1 = r$ ✓ $\quad \pi r^2(r) - 0.75r^3$

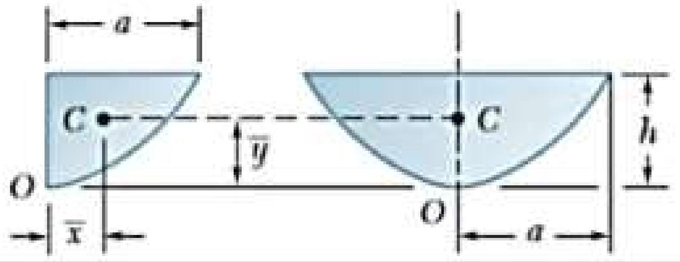
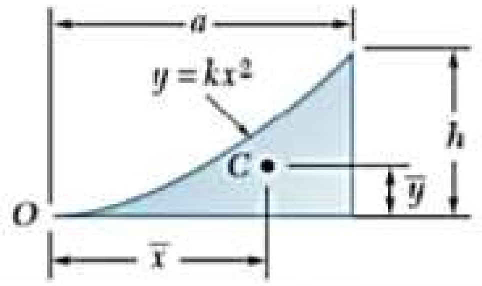
$A_2 = \frac{r^2}{2}$

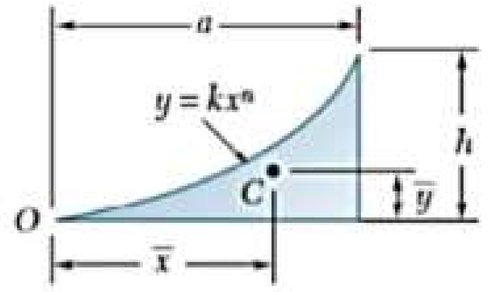
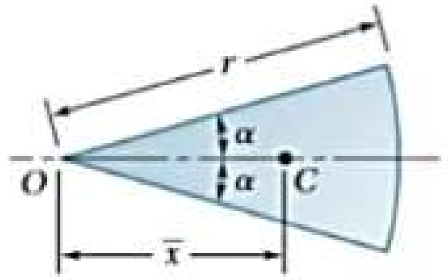
$\bar{x}_2 = \frac{3r}{2} = 1.5r$

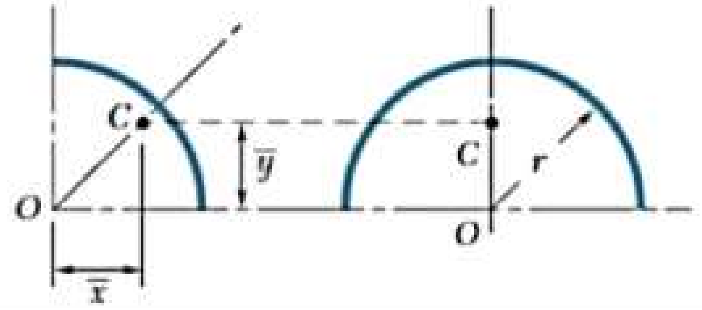
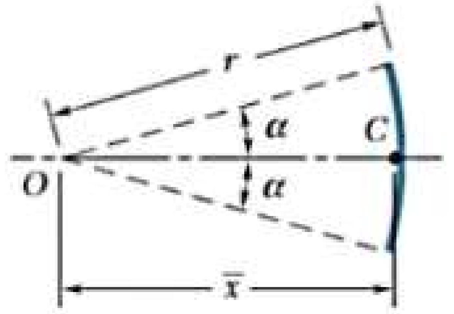




$\frac{r^2}{2}$

Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$

General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

$\bar{x}$ ✓ $\bar{y}$ ✓

$$\bar{x} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2 - A_3\bar{x}_3}{A_1 + A_2 - A_3} = 41.4\,mm$$

$\bar{y} =$ //

$A_1 \rightarrow 80 \times 30$

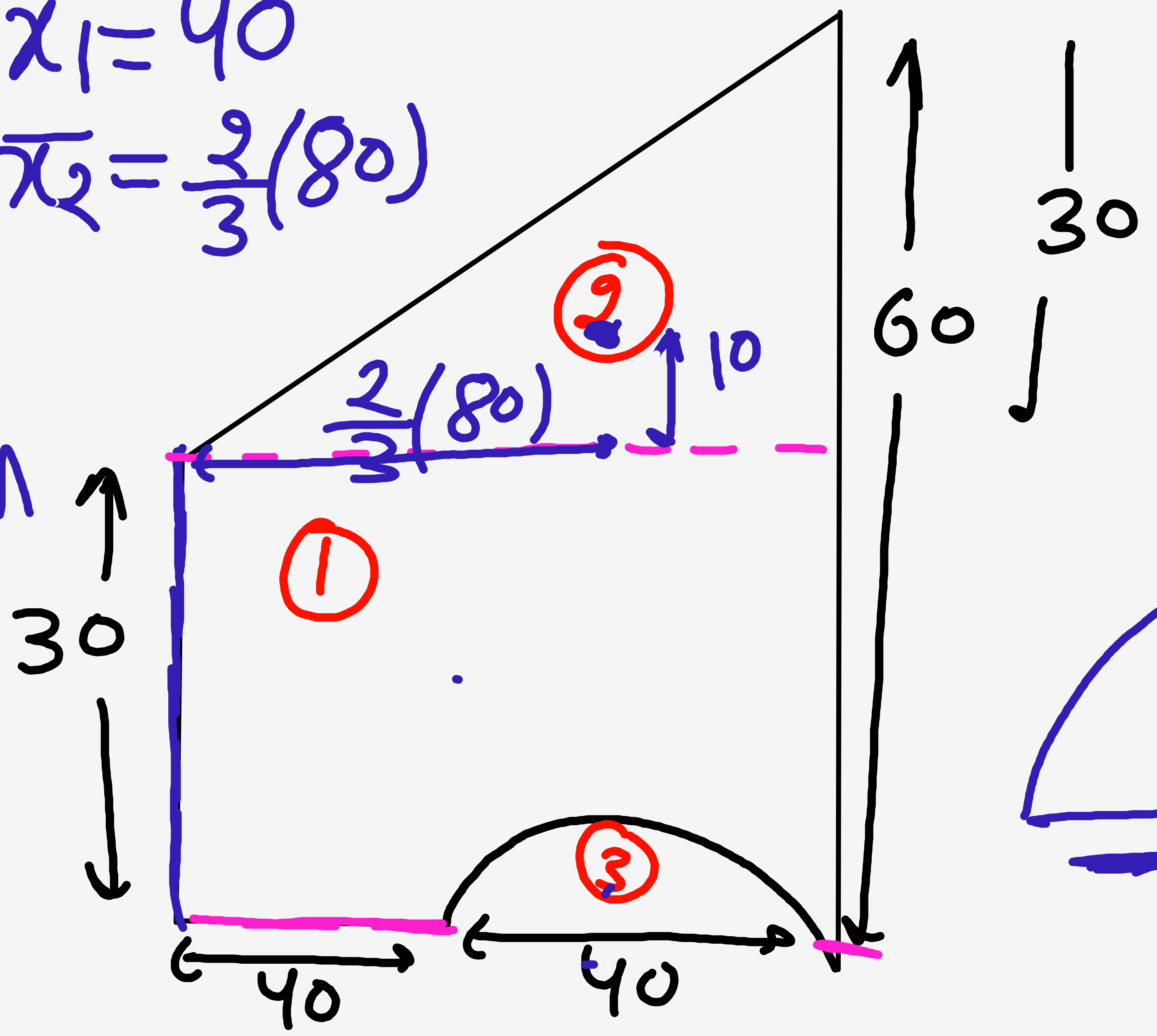
$A_2 \rightarrow \frac{1}{2} \times 80 \times 80$

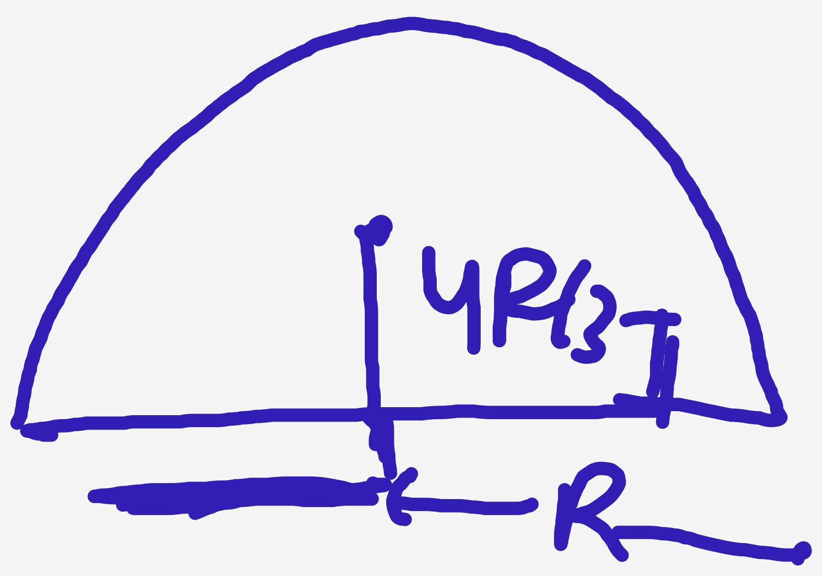
$A_3 \rightarrow \frac{\pi(20)^2}{2} = 628.3$

$\bar{y}_1 = 15$

$\bar{x}_1 = 40$

$\bar{x}_2 = \frac{2}{3}(80)$





$\bar{x}_3 = 60$

$\bar{y}_2 = 40$

$r = 20$

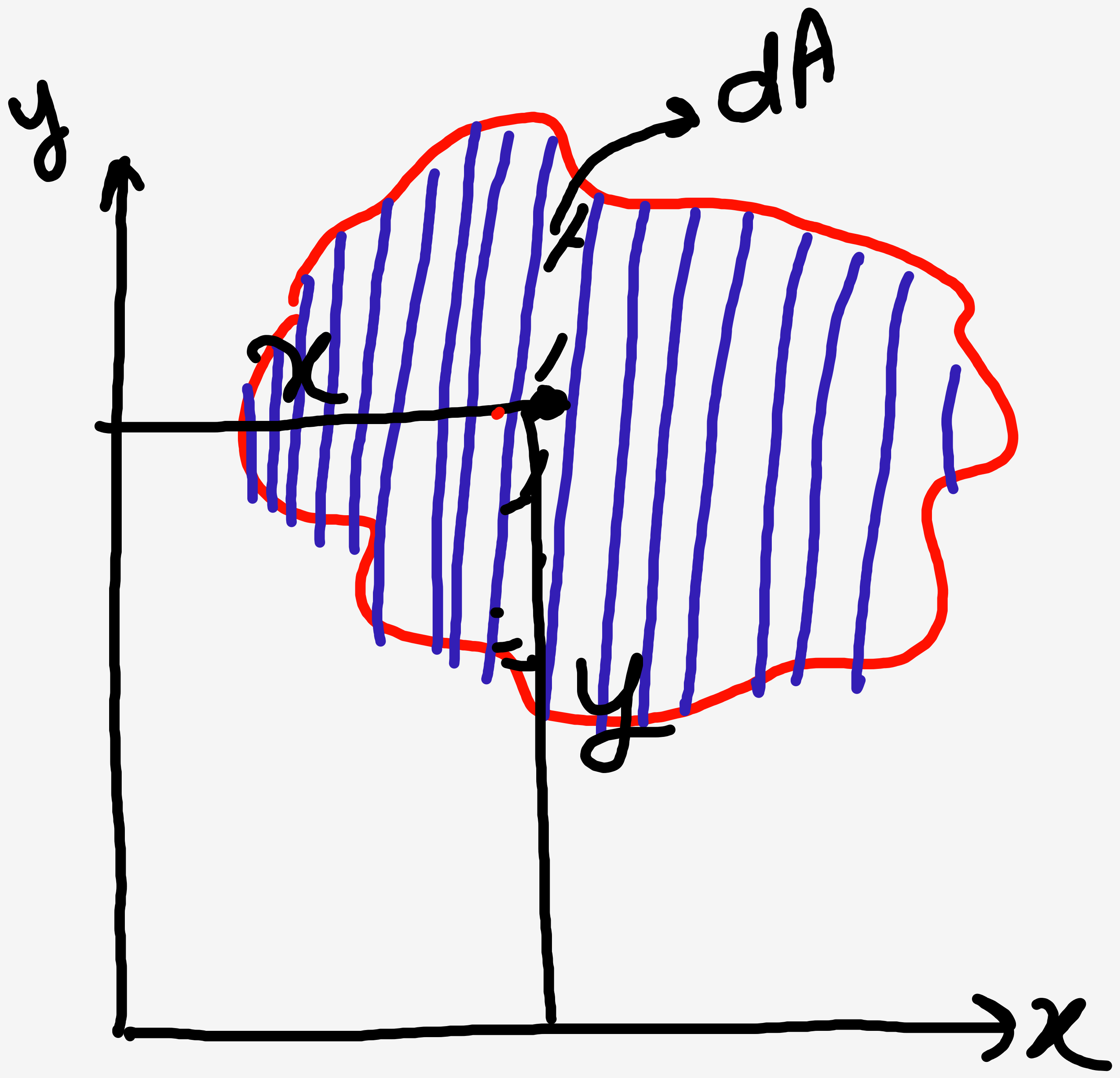
$\bar{y}_3 = \frac{4(20)}{3\pi} = 8.5\,mm$

MOMENT OF INERTIA

$$I_{xx} = \int y^2 \, dA$$

$$I_{yy} = \int x^2 \, dx$$

$$\int x^2 dx$$



$$\int dA \, y^2$$

RECTANGULAR X-S/C

$$\int_{y=-d/2}^{y=+d/2} dA \cdot y^2$$

$dA = b \cdot dy$

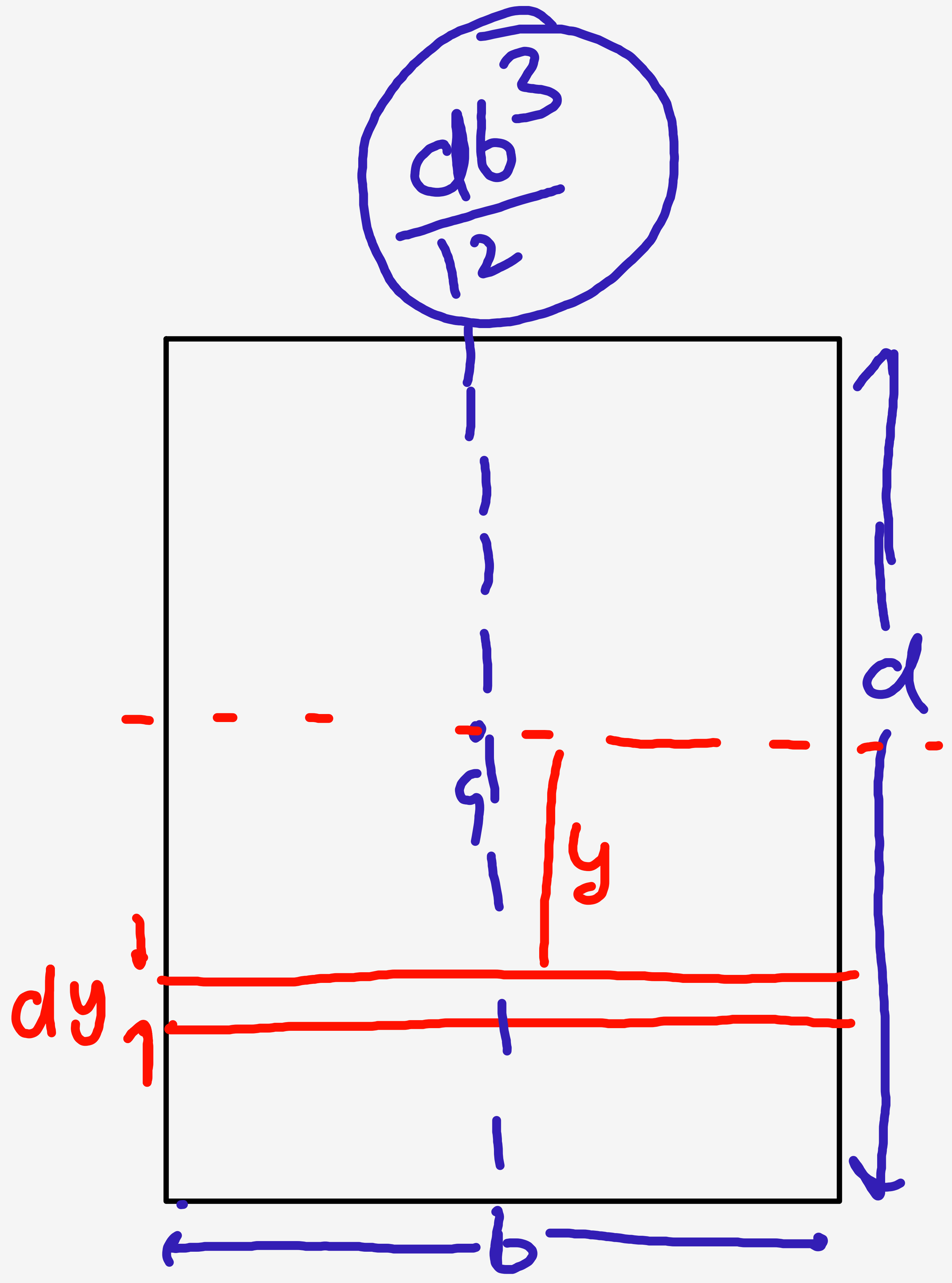
$$\int_{y=-d/2}^{y=+d/2} b\, y^2\, dy$$

$$b\left(\frac{y^3}{3}\right)_{-d/2}^{+d/2}$$

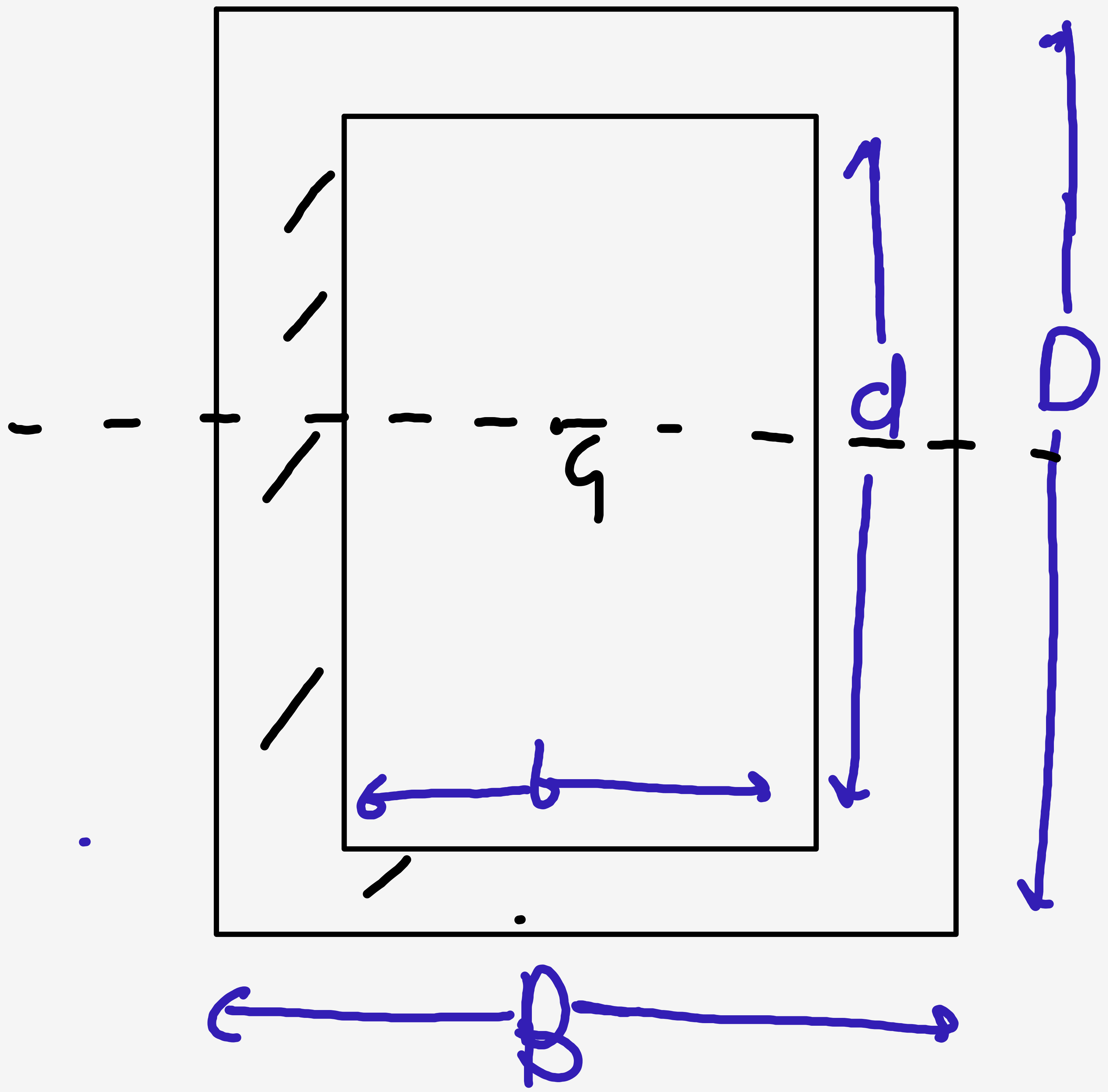
$$\frac{b}{3}\left[\left(\frac{d}{2}\right)^3 - \left(-\frac{d}{2}\right)^3\right]$$

$$2 \times \left(\frac{d}{2}\right)^3 \times \frac{b}{3}$$

$$\frac{bd^3}{12}$$

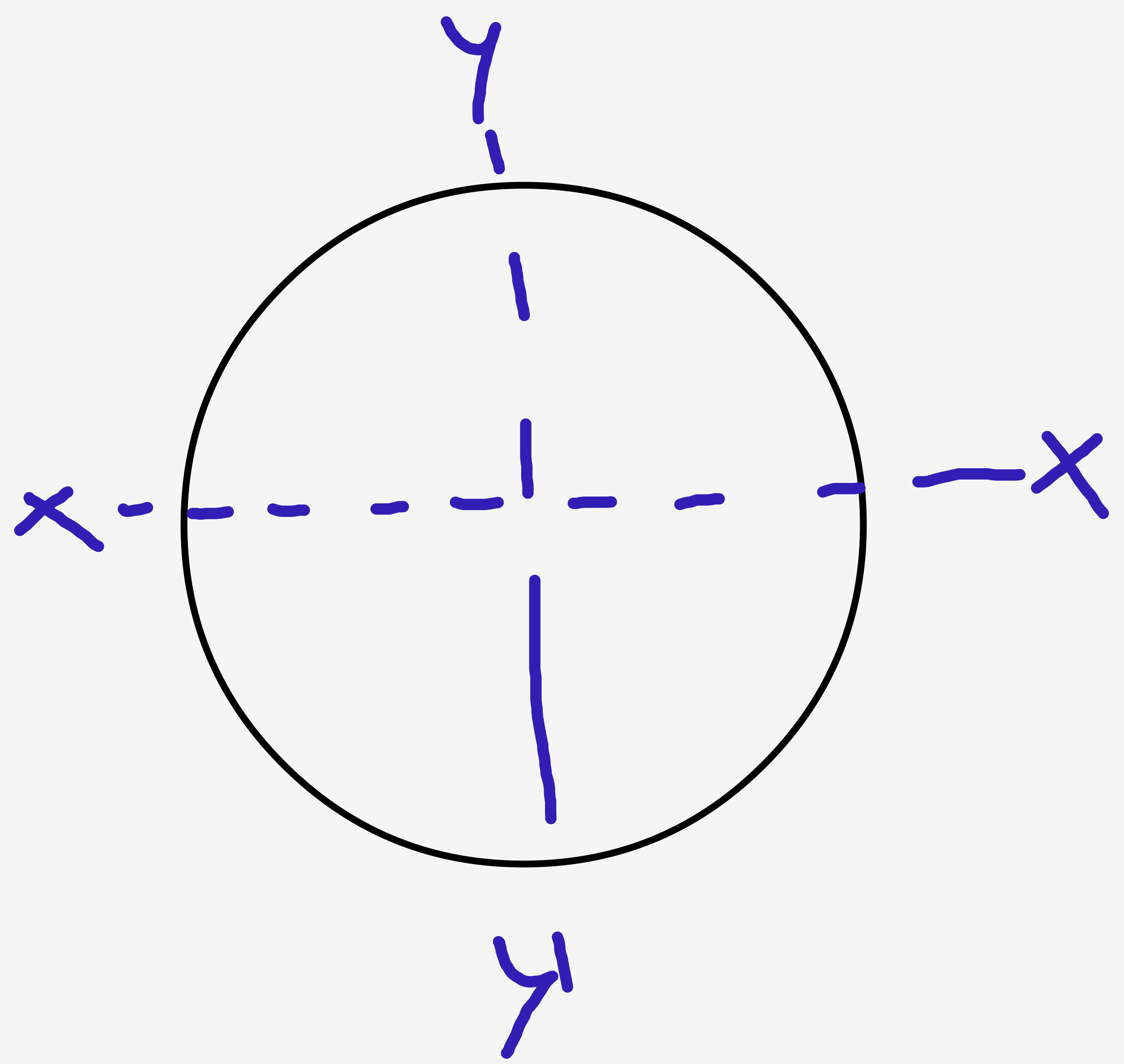


$$\frac{BD^3}{12} - \frac{bd^3}{12}$$



THEOREM OF PERPENDICULAR

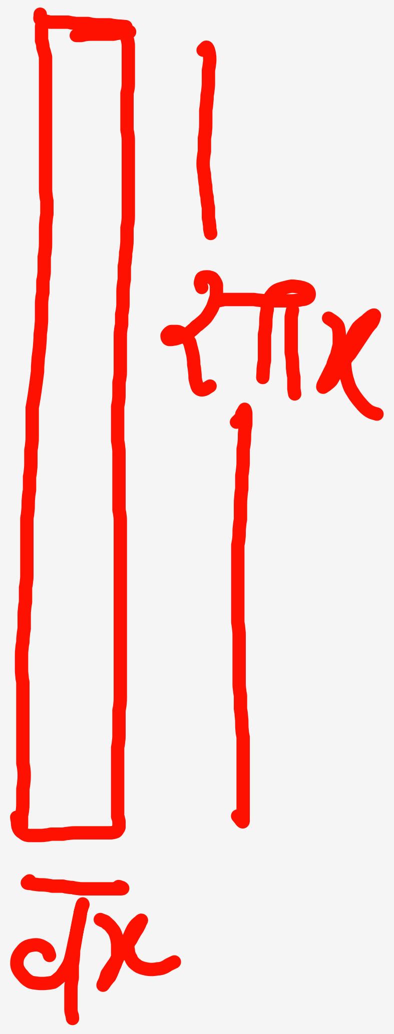
$$I_{xx} + I_{yy} = I_{zz}$$



MOI OF CIRCULAR SECTION

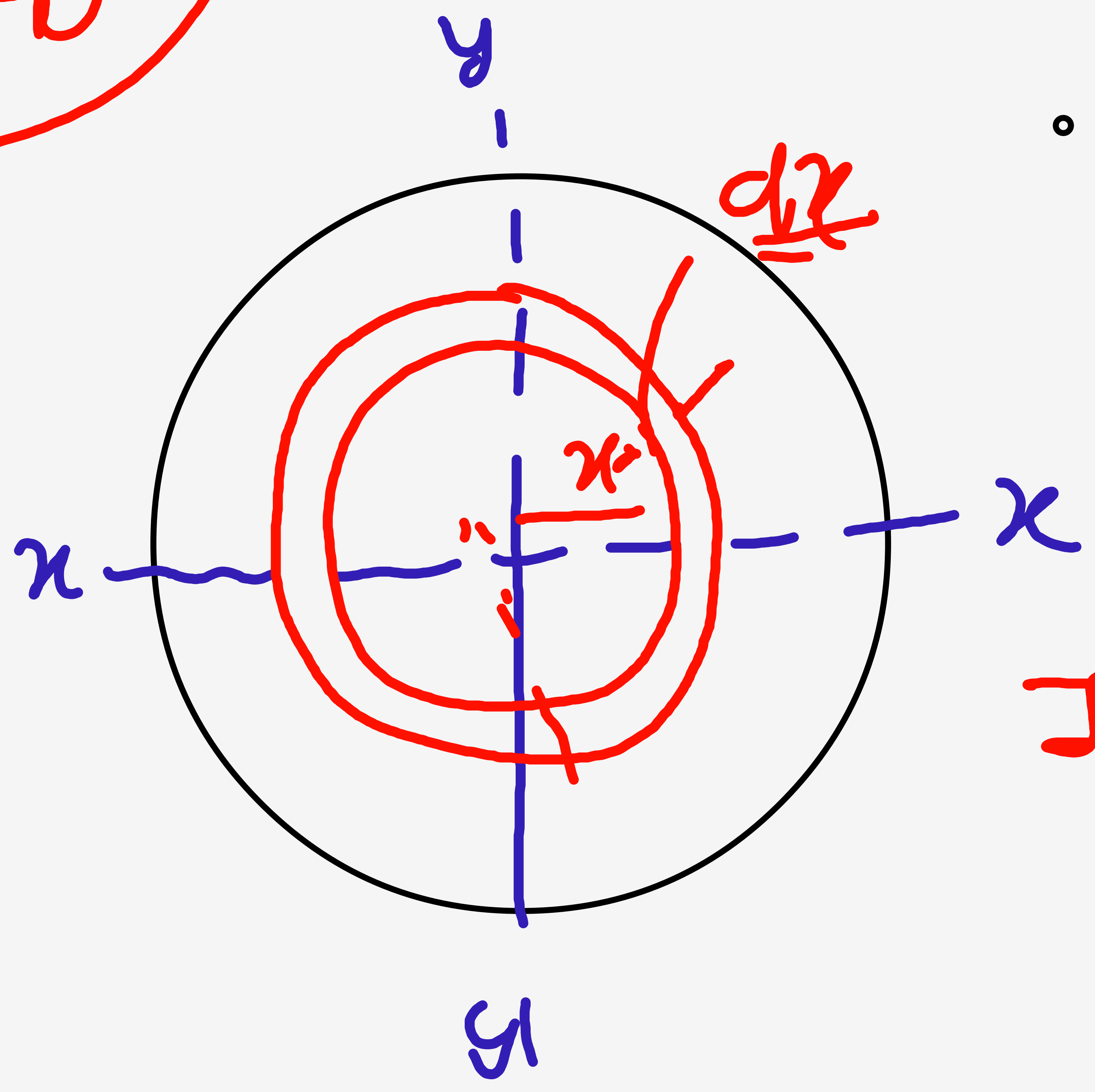
$R \to \frac{D}{2}$

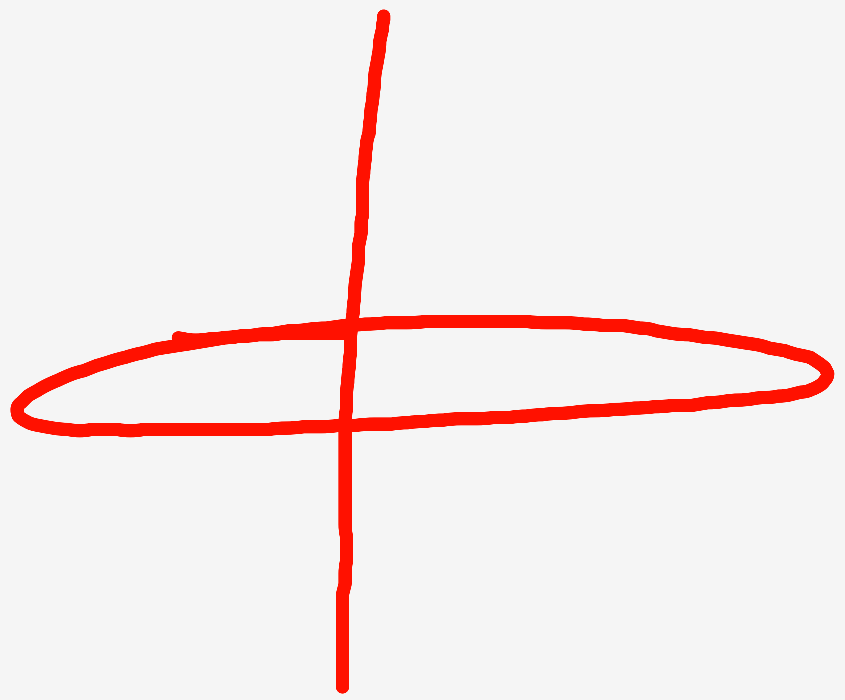
$\frac{\pi}{32} D^4$



$\frac{\pi}{2}(R^4)$

$\frac{\pi}{2}\left(\frac{D}{2}\right)^4$





$\int dA \cdot x^2$

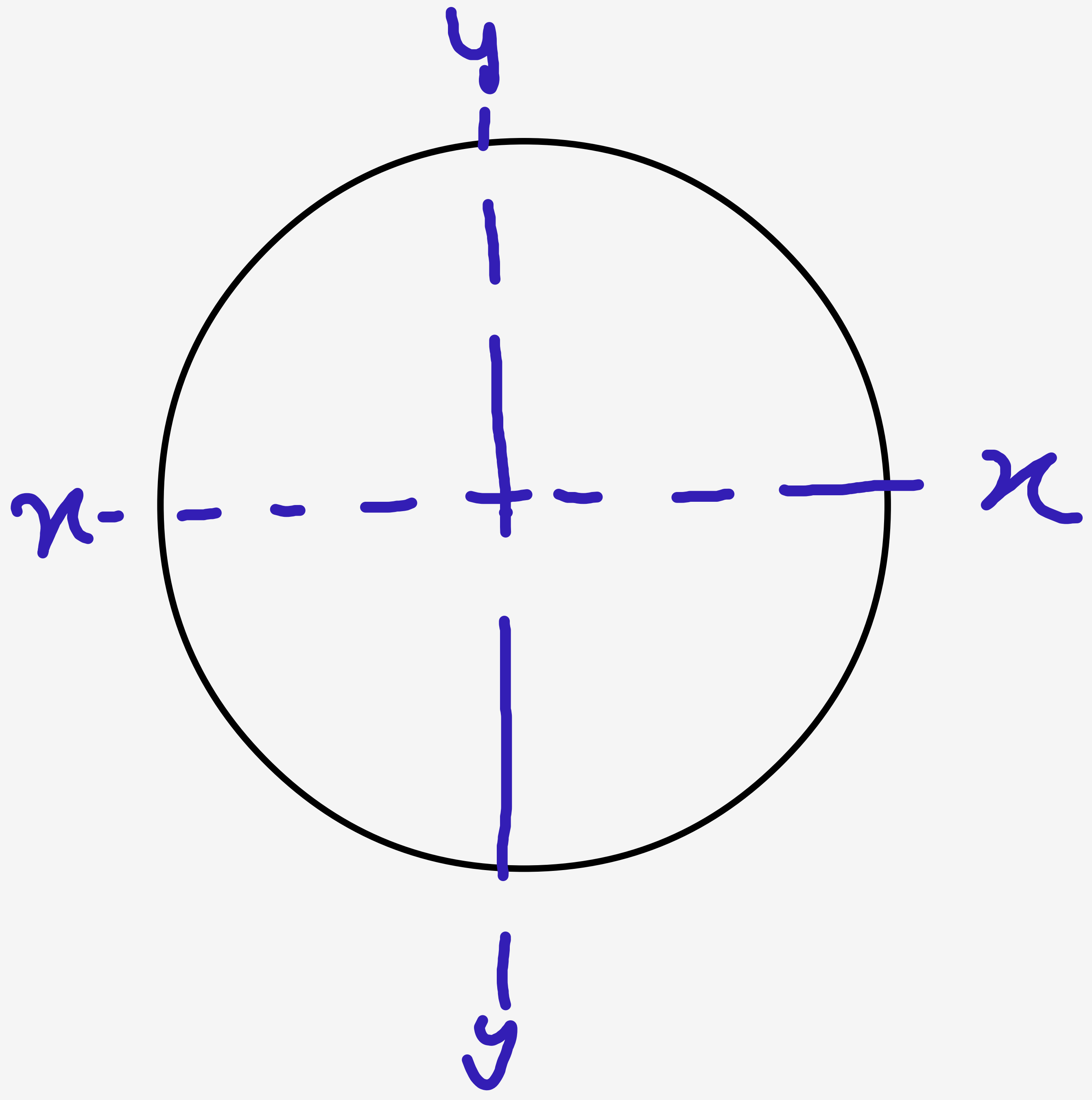
$I_{zz} = \int x^2 (2\pi x)\, dx$

$2\pi \left(\frac{x^4}{4}\right)\Big|_{x=0}^{x=R}$

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{xx} = \frac{\pi}{64} D^4$$

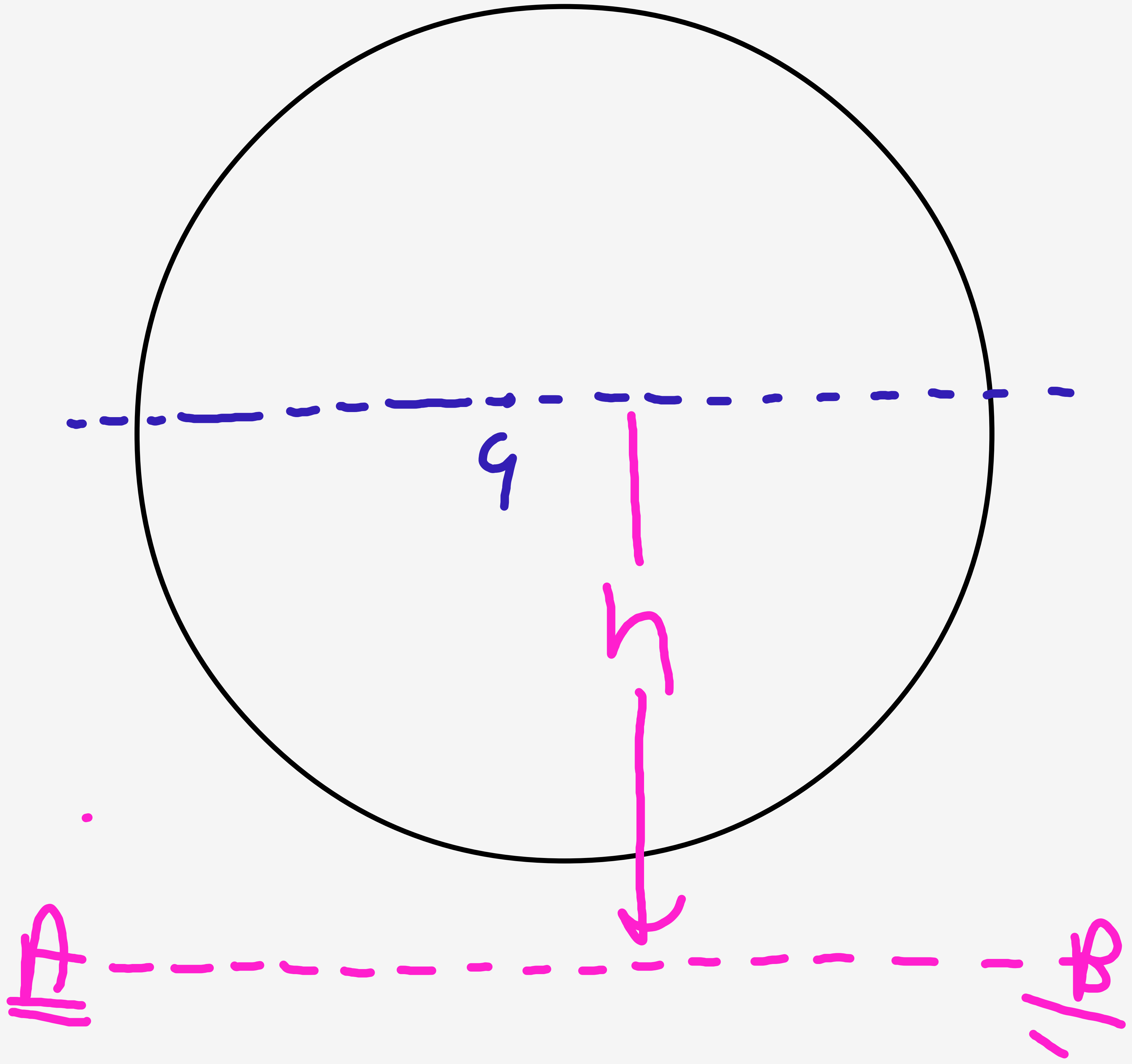
$$I_{yy} = \frac{\pi}{64} D^4$$

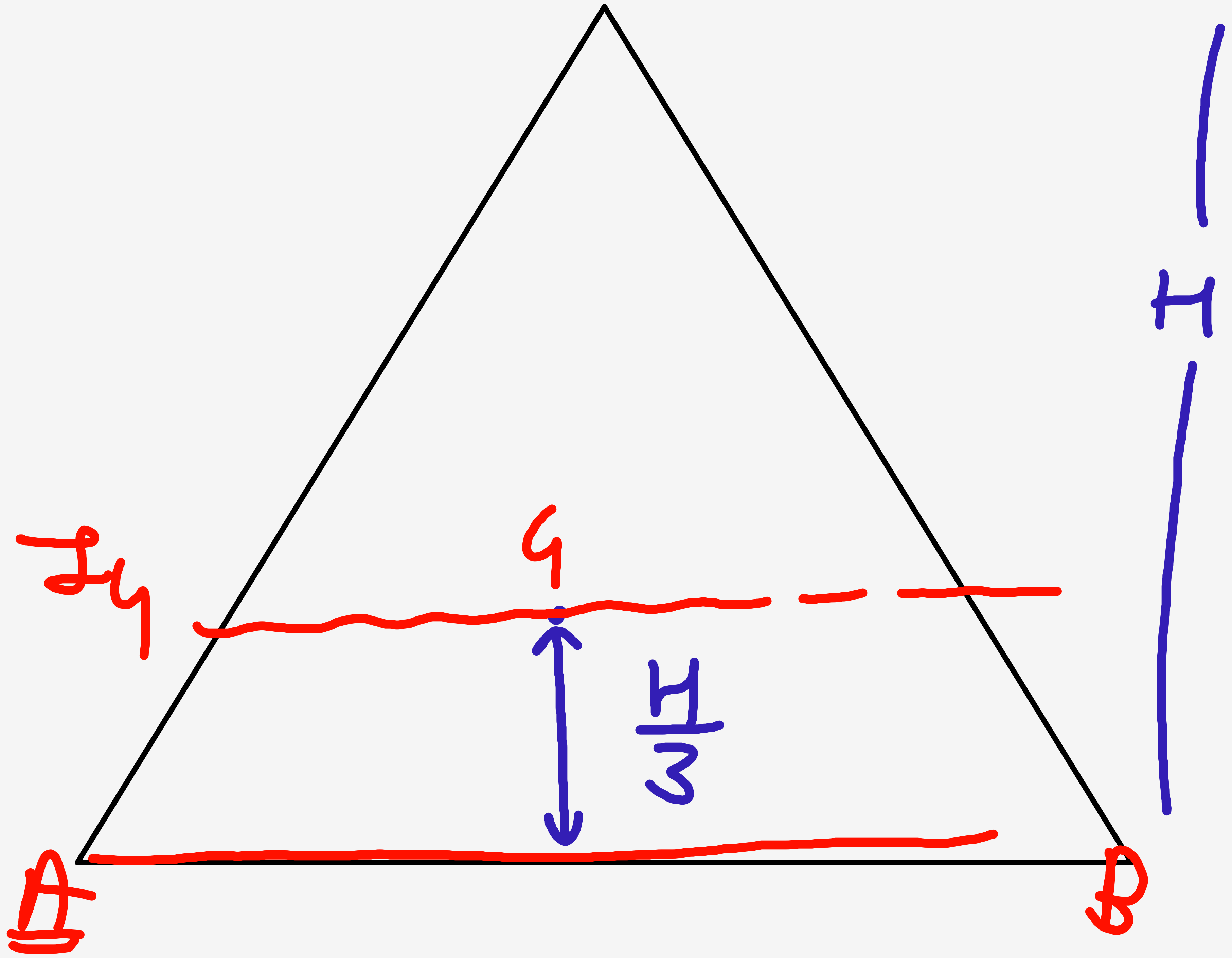


$$I_{zz} = \frac{\pi}{32} D^4$$

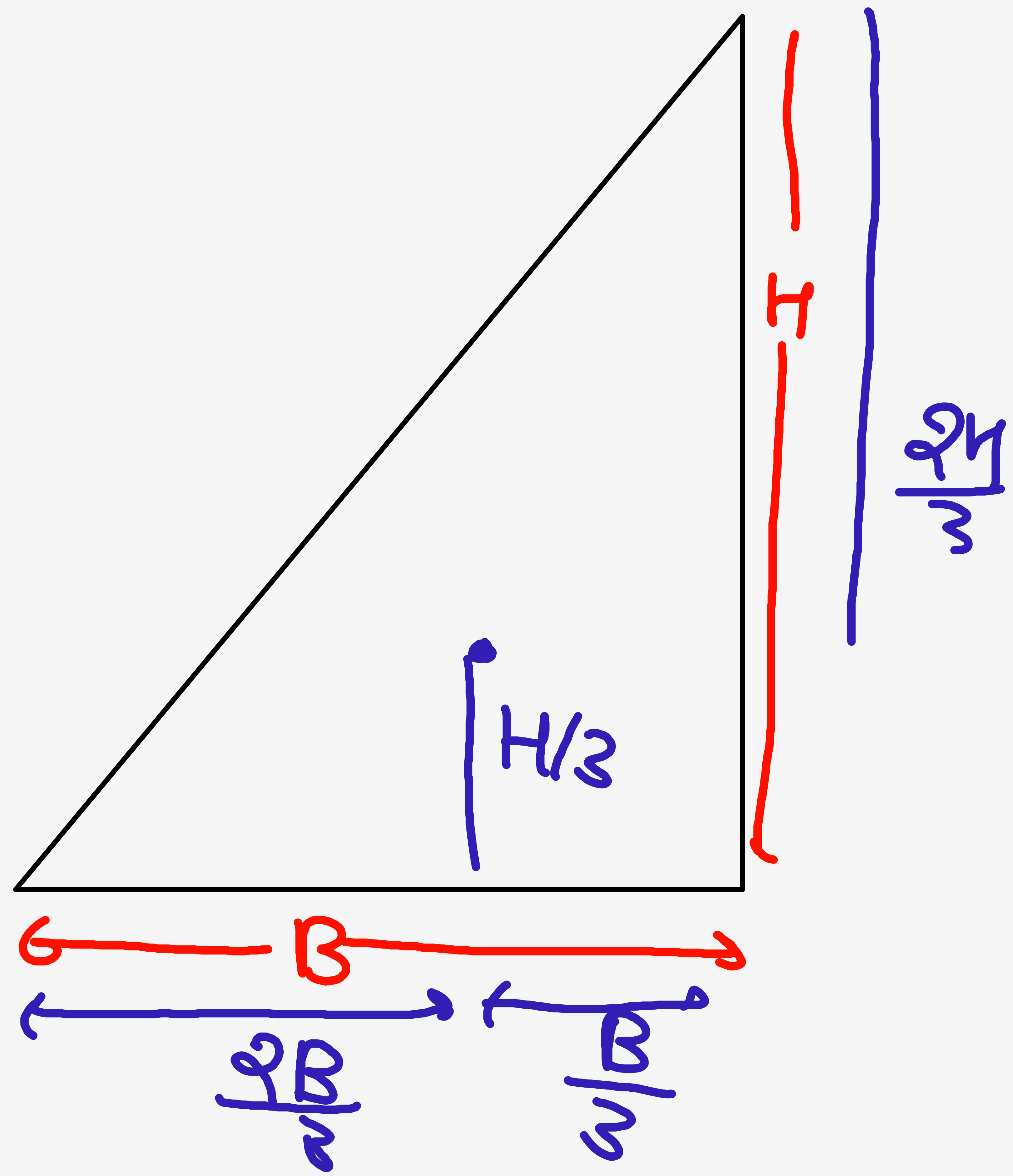
$\underline{I_g}$

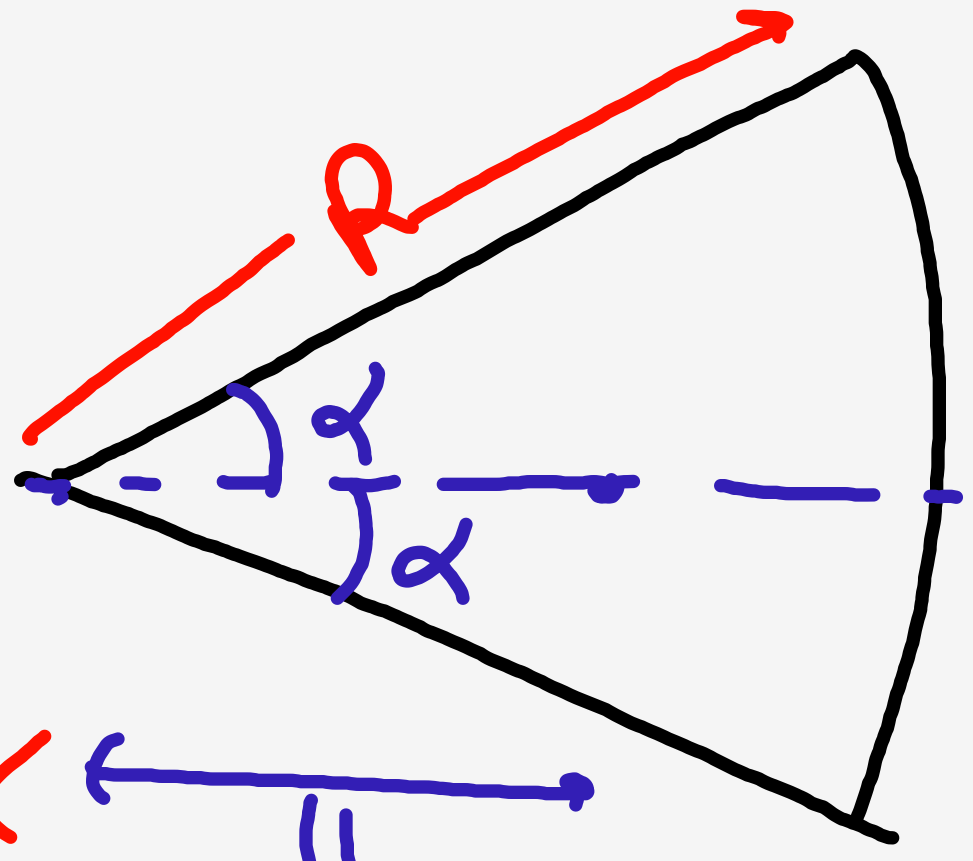
$$I_{AB} = I_g + Ah^2$$





$$I_{AB} = I_G + \frac{1}{2}B(H) \times \left(\frac{H}{3}\right)^2$$


H
2H/3
H/3
B
2B/3
B/3



$x \rightarrow \bar{y}$ की TENSION X ⟷

$\bar{x} =$

$$\bar{x} = \frac{2R\sin\alpha}{3\alpha}$$