Find sum of Coefficients & B.C./Combinatorial Coeff.

(i) $(4x+5y)^{100}$

(ii) $(3x-4y+5z)^{10}$

(i) $(4x+5y)^{100} = C_0 \cdot (4x)^{100} \cdot (5y)^0 + C_1 \cdot (4x)^{99} \cdot (5y)^1 + \dots + C_{100}(5y)^{100}$

$4x = 5y = 1$

$\underbrace{C_0 + C_1 + C_2 + \dots + C_{100}} = 2^{100}$

Sum of coeff $= 9^{100}$

$x=1,\ y=1$

(11)

$$(3x-4y+5z)^{10}$$

Sum of Combinatorial coeff. $= 3^{10}$

$(3x=1, -4y=1, 5z=1)$

Sum of coeff. $= (3-4+5)^{10} = \underline{4^{10}}$

$x=1, y=1, z=1$

(iii) $(5 + 2x - 3y + 10z)^{100}$

Sum of $\underbrace{\text{combinatorial coeff.}}$ $= 4^{100}$

Sum of coeff. $= (5+2-3+10)^{100} = \underline{14^{100}}$

$x=1,\ y=1,\ z=1$

$(5+x)^{100} = \underbrace{C_0} \cdot 5^{100} + \underbrace{C_1} \cdot 5^{99} x^1 + \underbrace{C_2} \cdot 5^{98} \cdot x^2 + \dots + C_{100} \cdot x^{100}$

(N)

$$(2 + x + 2y + 3z + 5k)^{10}$$

Sum of combinatorial coeff. $= 5^{10}$

Sum of coeff. $= (2 + 1 + 2 + 3 + 5)^{10} = 13^{10}$

$x = 1,\ y = 1,\ z = 1,\ k = 1$

Middle term

e.g.

(i) $(x+1)^{10}$ find middle term

(ii) $(x+1)^{11}$ " " "

(iii) $(x+1)^{100}$ " " "

(iv) $(x+1)^{101}$ " " "

Total terms = 101

$\underline{T_1 --- T_{50}}\ T_{51}\ T_{52} -- \underline{T_{101}}$

T_{51} & T_{52}

① $(x+1)^{10} = \underbrace{T_1 + T_2 + T_3 + - T_5}\ T_6\ \underbrace{T_7\ T_8\ T_9\ T_{10} + T_{11}}$

② $(x+1)^{11} = T_1\ T_2\ T_3\ T_4\ T_5\ \boxed{T_6\ |\ T_7}\ T_8\ T_9\ T_{10}\ T_{11}\ T_{12}$

$\underline{T_6\ \&\ T_7}$ are two middle terms

③

Note:- Binomial coeff. of middle term is greatest binomial coeff.

$$(x+y)^0 = 1$$

$$(x+y)^1 = x+y$$

$$(x+y)^2 = x^2+2xy+y^2$$

$$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$$

$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

find maximum value of ${}^{52}C_r$

${}^{52}C_0, {}^{52}C_1, {}^{52}C_2 - - - — \quad {}^{52}C_{52}$

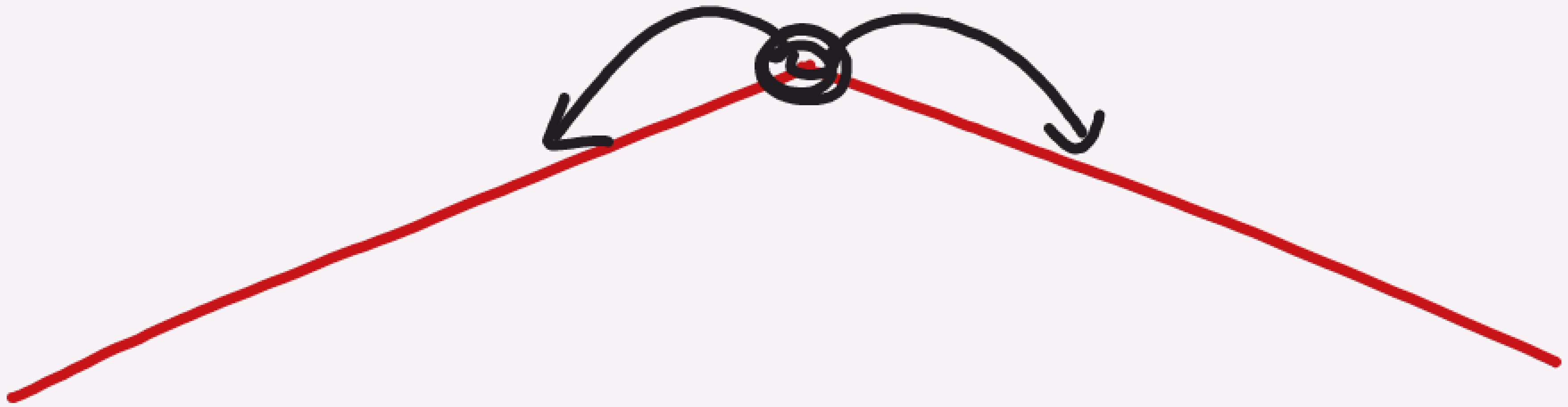
27^{th} coeff is max. ${}^{52}C_{26}$

${}^{n}C_r$ is max. if $\begin{cases} r = \frac{n}{2} & \text{when } n \in \text{Even no.} \\ r = \frac{n-1}{2} \text{ or } \frac{n+1}{2} & \text{when } n \in \text{odd no.} \end{cases}$

Numerically Greatest term

$$(x+y)^4 = \overset{1}{{}^4C_0}\cdot x^4 + \overset{4}{{}^4C_1}\cdot x^3\cdot y + \overset{6}{{}^4C_2}\cdot x^2y^2 + \overset{4}{{}^4C_3}\cdot xy^3 + \overset{1}{{}^4C_4}\cdot y^4$$

$(x=1, y=1)$ $\quad {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$



Let T_{r+1} is NGT

$$\therefore \quad |T_r| \le |T_{r+1}| \ge |T_{r+2}|$$

$$|T_{r+1}| \ge |T_r| \Rightarrow {}^nC_r\,|x^{n-r}\cdot y^r| \ge {}^nC_{r-1}\,|x^{n-r+1}\cdot y^{r-1}|$$

$$\Rightarrow \frac{\cancel{n!}}{\cancel{(n-r)!}\,r!}|y| \ge \frac{\cancel{n!}}{(n-r+1)!\,\cancel{(r-1)!}}|x| \Rightarrow \frac{|y|}{r} \ge \frac{|x|}{n-r+1}$$

$$\Rightarrow (n-r+1)|y| \ge |x|\cdot r \Rightarrow (n+1)|y| \ge r(|x|+|y|)$$

$$r \le \frac{(n+1)|y|}{|x|+|y|}$$

$$r \le \frac{(n+1)}{1+\left|\frac{x}{y}\right|}$$

Similarly by solving 2^{nd} part

$$r \ge \frac{n+1}{\left|\frac{x}{y}\right|+1} - 1$$

$$\boxed{\frac{n+1}{1+\left|\frac{x}{y}\right|} - 1 \qquad r \le \frac{n+1}{1+\left|\frac{x}{y}\right|}}$$

steps to find NGT

S-1. first find $k = \dfrac{n+1}{\left|\frac{x}{y}\right|+1}$

S-2. Solve for integral value of r satisfying

$$(k-1) \le r \le k$$

Q.1 If $x = \frac{1}{3}$ find NGT in $(1+4x)^8$

Q.2 If $x = 1$ find NGT in $(3-2x)^9$

Q.3 If T_6 is NGT in $\left(\frac{3}{2}+\frac{x}{3}\right)^n$ when $x = \frac{1}{2}$ find

HW.

Possible values of n

②

$$k = \frac{n+1}{\left|\frac{x}{y}\right|+1} = \frac{10}{\left|\frac{3}{2}\right|+1} = \frac{10}{5/2} = 4$$

$3 \le r \le 4$

$\therefore$ integral value of r

3 & 4

(1) $k = \dfrac{n+1}{\left|\frac{x}{y}\right|+1}$ $\qquad (1+4x)^8 \quad , x = \frac{1}{3}$

$$K = \frac{9}{\left|\frac{1}{4\cdot\frac{1}{3}}\right|+1} = \frac{9}{\frac{3}{4}+1} = \frac{36}{7} \approx 5.1$$

$4.1 \le r \le 5.1$ so, integral value of $\underline{r = 5}$

$\therefore$ NGT is $T_{r+1} = \underline{T_6}$